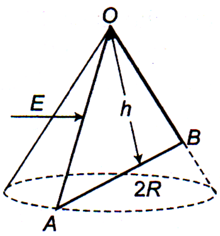
AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TARGET - JEE - TEST - 21
SOLUTION Date 06-10-2019

[PHYSICS]

1. **(b)** Plane normal to electric field is a triangle with base length $2R$ and height h.



Area of triangle $A = \frac{1}{2} \times 2Rh = Rh$

Electric flux entering the cone = $EA = ERh$

2. **(c)** Flux going in pyramid = $\frac{Q}{8\varepsilon_0}$

Which is divided equally among all 4 faces.

$\therefore$ Flux through one face = $\frac{Q}{8\varepsilon_0}$

3. **(b)** Total enclosed charge $q = 100\ Q$ coulomb

$$\phi_E = \frac{q}{\varepsilon_0} = \frac{100Q}{\varepsilon_0}$$

4. **(d)** Charge responsible for producing flux in the shell = $q/3$. In this case, angle formed by the shell at centre of ring $\frac{q}{3\varepsilon_0}$

So, charge to contribute flux in the shell

$$= \frac{2\pi}{3} \frac{q}{2\pi} = \frac{q}{3}$$

It is possible only if radii of ring and shell are equal and circumference of each passes through the centre of the other.

So, radius of ring = R

5. **(a)** Electric flux, $\phi = \dfrac{q}{\varepsilon_0}$

Where q = total charge enclosed by closed surface

$$\phi = \frac{1.25+7+1-0.4}{\varepsilon_0}$$

$$= \frac{8.85\text{C}}{8.85\times10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}}$$

$$= 10^{12}\ \text{N-m}^2/\text{C}$$

6. **(d)** By Gauss's law $\varphi = \dfrac{1}{\varepsilon_0}(Q_{\text{enclosed}})$

$$\Rightarrow Q_{\text{enclosed}} = \varphi\varepsilon_0 = (-8\times10^3 + 4\times10^3)\varepsilon_0$$

$$= -4\times10^3\varepsilon_0 \text{ Coulomb.}$$

7. **(c)** The electric field is due to all charges present whether inside or outside the given surface.

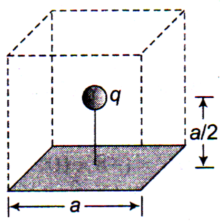
8. **(b)** The x-component of electric field is constant and does not contribute to net flux through the closed surface.
Due to y-component,
Incoming flux = –6 units ($\because y = 0$)
Outgoing flux = 9 units ($\because y = 1$)

By Gauss' Law, $\phi_{\text{net}} = \dfrac{q}{\varepsilon_0}$

$$\Rightarrow \quad (9-6) = \frac{q}{\varepsilon_0}$$

Hence, $q = 3\varepsilon_0$

9. **(d)** An imaginary cube can be made by considering charge q at the centre and given square is one of its face.



So flux from given square (i.e., one face) $\varphi = \dfrac{q}{6\varepsilon_0}$

10. **(b)** $\vec{E} = \dfrac{\sigma}{2\varepsilon_o}\hat{k} - \dfrac{2\sigma}{2\varepsilon_o}\hat{k} - \dfrac{\sigma}{2\varepsilon_o}\hat{k} = -\dfrac{2\sigma}{\varepsilon_o}\hat{k}$

11. **(b)** $E_{\text{inside}} = \dfrac{\rho}{3\varepsilon_0} r \quad (r < R)$

$E_{\text{outside}} = \dfrac{\rho R^3}{3\varepsilon_0 r^2} \quad (r \geq R)$

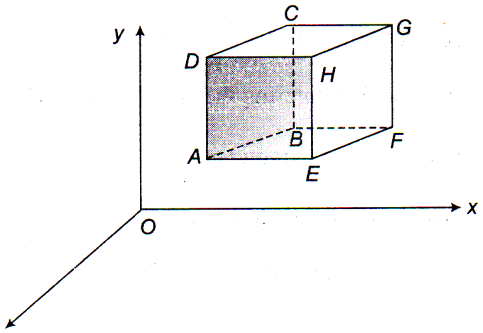
i.e., inside the uniformly charged sphere field varies linearly ($E \propto r$) with distance and outside varies according to $E \propto \dfrac{1}{r^2}$

12. **(b)** Field at any point inside the cavity is uniform and non-zero. Hence, option (b).

13. **(b)** Field at face $ABCD = E_0 x_0 \hat{i}$

Flux over the face $ABCD = -E_0 x_0 a^2$

The negative sign arises as the field is directed ir the cube.



Field at face $EFGH = E_0(x_0 + a)\hat{i}$

Flux over the face $EFGH = E_0(x_0 + a)a^2$

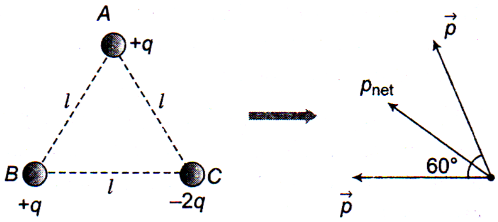
Flux over the other four faces is zero as the field is parallel to the surfaces.

Total flux over the cube $= E_0 a^3 = \dfrac{1}{\varepsilon_0} q$.

where q is the total charge under the cube

$\therefore\ q = \varepsilon_0 E_0 a^3$.

14. **(c)**



$p_{\text{net}} = \sqrt{p^2 + p^2 + 2pp\cos 60^\circ} = \sqrt{3}p = \sqrt{3}\,ql$

$(\because p = ql)$

15. **(b)** We have $E_a = \frac{2kp}{r^3}$ and $E_e = \frac{kp}{r^3}$; $\therefore\ E_a = 2E_e$

16. **(d)** $E_{\text{axial}} = E_{\text{equatorial}} \Rightarrow k.\frac{2p}{x^3} = \frac{k.p}{y^3} \Rightarrow \frac{x}{y} = \frac{2^{1/3}}{1} = \sqrt[3]{2} : 1$

17. **(c)** $\tau_{\max} = pE = q(2l)E = 2\times10^{-6}\times0.01\times5\times10^5$

$= 10\times10^{-3}\,\text{N-m}$

18. **(c)** There are 10 electrons and 10 protons in a neutral water molecule.

So its dipole moment is $p = q\,(2l) = 10\,e\,(2l)$

Hence length of the dipole i.e., distance between centres of positive and negative charges is

$$2l = \frac{p}{10e} = \frac{6.4\times10^{-20}}{10\times1.6\times10^{-19}} = 4\times10^{-12}\,\text{m} = 4\text{ pm}$$

19. **(b)** Here, 2a = 2 cm = 0.02 m, $\tau_{\max} = 0.2\times10^{-3}\,\text{N m}$

and $E = 10^5\,\text{N C}^{-1}$

Now, maximum torque on the dipole,

$\tau_{\max} = pE = q(2a)\times E$

$$\therefore\quad q = \frac{\tau_{\max}}{(2a)E} = \frac{0.2\times10^{-3}}{0.02\times10^5} = 10^{-7}C = 10\,\mu\text{C}$$

20. **(c)** Point P lies at equatorial positions of dipole 1 and 2 and axial position of dipole 3.

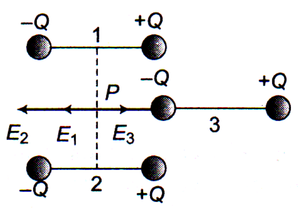
Hence field at P due to dipole 1

$E_1 = \frac{k.p}{x^3}$ (towards left) due to dipole 2

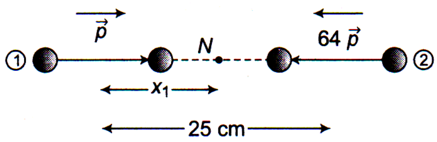
$E_2 = \frac{k.p}{x^2}$ (towards left) due to dipole 3 $E_3 = \frac{k(2p)}{x^3}$

(towards right)

So net field at P will be zero.



21. **(a)** Suppose neutral point N lies at a distance x from dipole of moment p or at a distance x_2 from dipole of $64p$.



At N |E. F. due to dipole ① = |E. F. due to dipole ②|

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{x^3} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2(64p)}{(25-x)^3}$$

22. **(a)** When dipole is given a small angular displacement θ about its equilibrium position, the restoring torque will be

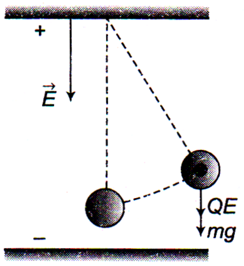
$\tau = -pE\sin\theta = -pE\theta$ (as $\sin\theta = \theta$)

or $I\frac{d^2\theta}{dt^2} = -pE\theta$ (as $\tau = I\alpha = I\frac{d^2\theta}{dt^2}$)

or $\frac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \frac{pE}{I} \Rightarrow \omega = \sqrt{\frac{pE}{I}}$

23. **(d)** Electric field due to ring and dipole moment are parallel to each other.

24. **(c)**

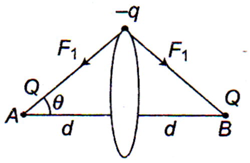


Net downward force $mg' = mg + QE$

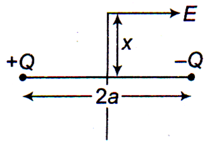
$\Rightarrow$ Effect acceleration $g' = \left(g + \frac{QE}{m}\right)$

Hence time period $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\left(g + \frac{QE}{m}\right)}}$

25. **(b)** Net force on $-q$ towards the centre,



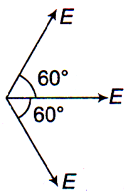
26. **(b)** Two opposite charges situated at opposite vertices forms a dipole and point lies on bisector line of dipole for one dipole



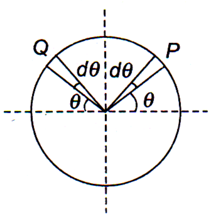
$$\vec{E} = \frac{-K\vec{P}}{x^3}$$

direction of field due to these three dipoles is in a horizontal plane at distance x from plane of hexagon and parallel to it, mutually having angle 60° with each other.

$$E_0 = E + E = 2E = 2\frac{Q\,(2a)}{4\pi\varepsilon_0 . x^3} = \frac{Qa}{\pi\,\varepsilon_0 x^3}.$$



27. **(d)** Dipole moment of the charge pair at P and Q.



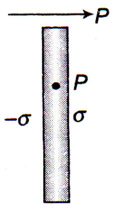
$$dp = \left[(\lambda_0 \cos\theta)Rd\theta\right](2R\cos\theta)$$

Total dipole moment

$$= 2\lambda_0 R^2 \int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta = \pi R^2 \lambda_0$$

28. **(b)** Net electric field at point P should be zero. For this electric field due to induced charges = applied electric field

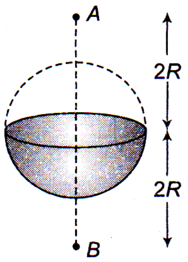
$$\Rightarrow \frac{\sigma}{\varepsilon_0} = E \Rightarrow \quad \sigma = \varepsilon_0 E$$



29. **(b)** Let us complete the sphere. Electric field due to lower part at A = electric field due to upper part at $B = E$ (given)

Electric field due to lower part at B = electric field due to full sphere – electric field due to upper part

$$= \frac{kQ}{(2R)^2} - E = \frac{1}{4\pi\varepsilon_0}\frac{\rho(4/3)\pi R^3}{4R^2} - E = \frac{\rho R}{12\varepsilon_0} - E$$



30. **(a)** If portion outside the cylinder is removed, electric field at points on curved surface will decrease.

[MATHEMATICS]

61. (d) Let $f(x) = 2x^3 - 24x + 107$

At $x = -3, f(-3) = 2(-3)^3 - 24(-3) + 107 = 125$

At $x = 3, \ f(3) = 2(3)^3 - 24(3) + 107 = 89$

For maxima or minima, $f'(x) = 6x^2 - 24 = 0$

$\Rightarrow x = 2, \ -2$

So at $x = 2, f(2) = 2(2)^3 - 24(2) + 107 = 75$

at $x = -2, \ f(-2) = 2(-2)^3 - 24(-2) + 107 = 139$

Thus the maximum value of the given function in [– 3, 3] is 139.

62. (c) Given $f(x)=\frac{[(5+x)(2+x)]}{[1+x]}$

$f(x)=1+\frac{4}{1+x}+(5+x)=(6+x)+\frac{4}{(1+x)}$

$\Rightarrow f'(x)=1-\frac{4}{(1+x)^2}=0$; $x^2+2x-3=0 \Rightarrow x=-3, 1$

Now $f''(x)=\frac{8}{(1+x)^3}$, $f''(-3)=-ve$, $f''(1)=+ve$

Hence minimum value at $x=1$

$f(1)=\frac{(5+1)(2+1)}{(1+1)}=\frac{6\times 3}{2}=9$.

63. (c) $a^2x^4+b^2y^4=c^6 \Rightarrow y=\left(\frac{c^6-a^2x^4}{b^2}\right)^{1/4}$

Hence $f(x)=xy=x\left(\frac{c^6-a^2x^4}{b^2}\right)^{1/4}$

$\Rightarrow f(x)=\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{1/4}$

Differentiate $f(x)$ with respect to x, then

$f'(x)=\frac{1}{4}\left(\frac{c^6x^4-a^2x^8}{b^2}\right)^{-3/4}\left(\frac{4x^3c^6}{b^2}-\frac{8x^7a^2}{b^2}\right)$

Put $f'(x)=0$, $\frac{4x^3c^6}{b^2}-\frac{8x^7a^2}{b^2}=0$

$\Rightarrow x^4=\frac{c^6}{2a^2} \Rightarrow x=\pm\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$

At $x=\frac{c^{3/2}}{2^{1/4}\sqrt{a}}$ the $f(x)$ will be maximum, so

$f\left(\frac{c^{3/2}}{2^{1/4}\sqrt{a}}\right)=\left(\frac{c^{12}}{2a^2b^2}-\frac{c^{12}}{4a^2b^2}\right)^{1/4}=\left(\frac{c^{12}}{4a^2b^2}\right)^{1/4}=\frac{c^3}{\sqrt{2ab}}$.

64. (b) $y=f(x)=-x^3+3x^2+9x-27$

The slope of this curve $f'(x)=-3x^2+6x+9$

Let $g(x)=f'(x)=-3x^2+6x+9$

Differentiate with respect to x, $g'(x)=-6x+6$

Put $g'(x)=0 \Rightarrow x=1$

Now, $g''(x)=-6<0$ and hence at $x=1$, $g(x)$ (slope) will have maximum value.

$\therefore [g(1)]_{\text{max.}}=-3\times 1+6+9=12$.

65. (d) $f(x)=|px-9|+r|x|, x\in(-\infty, \infty)$

Where $p>0$, $q>0$ and $r>0$ can assume its minimum value only at one point, if $p=q=r$.

66. (a) Let co-ordinate of $R\ (x, 0)$
Given $P(1,1)$ and $Q(3,2)$

$$PR+RQ=\sqrt{(x-1)^2+(0-1)^2}+\sqrt{(x-3)^2+(0-2)^2}$$
$$=\sqrt{x^2-2x+2}+\sqrt{x^2-6x+13}$$

For minimum value of $PR + RQ$, $\frac{d}{dx}(PR+RQ)=0$

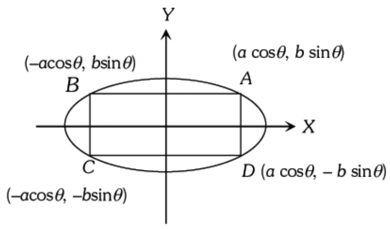
$$\Rightarrow \frac{d}{dx}(\sqrt{x^2-2x+2})+\frac{d}{dx}(\sqrt{x^2-6x+13})=0$$
$$\Rightarrow \frac{(x-1)}{\sqrt{x^2-2x+2}}=-\frac{(x-3)}{\sqrt{x^2-6x+13}}$$

Squaring both sides, $\frac{(x-1)^2}{(x^2-2x+2)}=\frac{(x-3)^2}{x^2-6x+13}$

$\Rightarrow 3x^2-2x-5=0 \Rightarrow (3x-5)(x+1)=0$, $x=\frac{5}{3},-1$.

Also $1<x<3$. $\therefore$ $R=(5/3,0)$.

67. (c)



Area of rectangle $ABCD$
$=(2a\cos\theta).(2b\sin\theta)=2ab\sin 2\theta$
Hence, area of greatest rectangle is equal to $2ab$, when $\sin 2\theta=1$.

68. (b) To be increasing $f'(x)=3x^2-27>0$
$\Rightarrow x^2>9 \Rightarrow |x|>3$.

69. (b) Since $f(x)=\sqrt{3}\sin x-\cos x-2ax+b$ is decreasing for all real values of x, therefore $f'(x)<0$ for all x.
$\Rightarrow \sqrt{3}\cos x+\sin x-2a<0$ for all x
$\Rightarrow \frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x<a$ for all x
$\Rightarrow \sin\left(x+\frac{\pi}{3}\right)<a$ for all $x \Rightarrow a\ge 1, \left[\because \sin\left(x+\frac{\pi}{3}\right)\le 1\right]$.

70. (c) The function $f(x)=x^3$ increases for all x and the function $g(x)=6x^2+15x+5$ increases, if
$g'(x)>0 \Rightarrow 12x+15>0 \Rightarrow x>-\frac{5}{4}$.
Thus $f(x)$ and $g(x)$ both increases for $x>-\frac{5}{4}$.

It is given that $f(x)$ increases less rapidly than $g(x)$,
Therefore the function $\phi(x)=f(x)-g(x)$ is decreasing function, which implies that $\phi'(x)<0$
$\Rightarrow 3x^2-12x-15<0 \Rightarrow -1<x<5$.

71. (d) If $f(x)=(a+2)x^3-3ax^2+9ax-1$ decreases monotonically for all $x \in R$, then $f'(x) \le 0$ for all $x \in R$

$\Rightarrow 3(a+2)x^2-6ax+9a \le 0$ for all $x \in R$

$\Rightarrow (a+2)x^2-2ax+3a \le 0$ for all $x \in R$

$\Rightarrow a+2<0$ and Discriminant ≤ 0

$\Rightarrow a<-2, -8a^2-24a \le 0 \Rightarrow a<-2$ and $a(a+3) \ge 0$

$\Rightarrow a<-2,\ a \le -3$ or $a \ge 0 \Rightarrow a \le -3 \Rightarrow -\infty < a \le -3$.

72. (c) $f'(x)=\dfrac{\sin x - x\cos x}{\sin^2 x}=\dfrac{\cos x(\tan x - x)}{\sin^2 x}$

$0<x\le 1 \Rightarrow x \in Q_1 \Rightarrow \tan x > x,\ \cos x > 0$

$\therefore f'(x)>0$ for $0<x\le 1$

$\therefore f(x)$ is an increasing function.

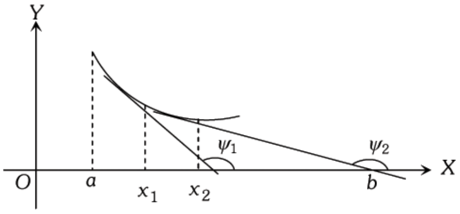
$g'(x)=\dfrac{\tan x - x\sec^2 x}{\tan^2 x}=\dfrac{\sin x\cos x - x}{\sin^2 x}=\dfrac{\sin 2x-2x}{2\sin^2 x}$

$(\sin 2x-2x)'=2\cos 2x-2=2[\cos 2x-1]<0$

$\Rightarrow \sin 2x-2x$ is decreasing $\Rightarrow \sin 2x-2x<0$

$\therefore g'(x)<0 \Rightarrow g(x)$ is decreasing.

73. (d) From the trend of value of $\sin x$ and $\cos x$ we know $\sin x$ and $\cos x$ decrease in $\frac{\pi}{2}<x<\pi$. So, the statement S is correct.



The statement R is incorrect which is clear from graph. Clearly $f(x)$ is differentiable in (a, b).

Also, $a<x_1<x_2<b$.

But $f'(x_1)=\tan\phi_1<\tan\phi_2=f'(x_2)$.

74. (d) Given $f(x)=x^3+bx^2+cx+d$

$\therefore f'(x)=3x^2+2bx+c$

Now its discriminant $=4(b^2-3c)$

$\Rightarrow 4(b^2-c)-8c<0$, as $b^2<c$ and $c>0$

Therefore, $f'(x)>0$ for all $x \in R$

Hence f is strictly increasing.

75. (a) $f(x)=3x^2-2x+1$, $f'(x)=6x-2\ge 0 \Rightarrow x \ge \frac{1}{3}$

Option (a) is incorrect. Checking other function similarly we find that they are correctly matched

76.

$y = x + 1$ will be || to tangent at $P(y_0^2, y_0)$ on $x = y^2$

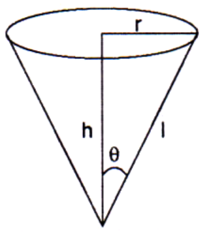
$\Rightarrow \dfrac{1}{2y_0} = 1 \Rightarrow y_0 = \dfrac{1}{2}$

$\Rightarrow P\left(\dfrac{1}{4}, \dfrac{1}{2}\right)$

$\Rightarrow$ ⊥r distance of line from P ≡ $\dfrac{\left|\frac{1}{4} - \frac{1}{2} + 1\right|}{\sqrt{(1)^2 + (-1)^2}} = \dfrac{3/4}{\sqrt{2}} = \dfrac{3}{4\sqrt{2}} = \dfrac{3\sqrt{2}}{8}$

77.

$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi r^2\left(\sqrt{\ell^2 - r^2}\right)$



$\therefore \dfrac{dV}{dr} = \dfrac{\pi}{3}\left[r^2\left(\dfrac{-2r}{2\sqrt{\ell^2 - r^2}}\right) + 2r\sqrt{\ell^2 - r^2}\right]$

$= \dfrac{\pi}{3}\left[\dfrac{-r^3 + 2r\left(\ell^2 - r^2\right)}{\sqrt{\ell^2 - r^2}}\right] = \dfrac{\pi}{3}\left[\dfrac{2r\ell^2 - 3r^3}{\sqrt{\ell^2 - r^2}}\right]$

$\Rightarrow$ For maximum volume $\dfrac{dV}{dr} = 0$

$\Rightarrow r(2\ell^2 - 3r^2) = 0$

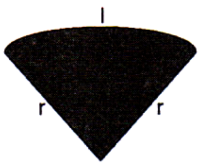
$\Rightarrow \dfrac{\ell^2}{r^2} = \dfrac{3}{2} \Rightarrow \dfrac{\ell}{r} = \sqrt{\dfrac{3}{2}}$

$\Rightarrow$ cosec θ = $\sqrt{3/2}$

$\Rightarrow$ cot θ = $\dfrac{1}{\sqrt{2}} \Rightarrow \tan\theta = \sqrt{2}$

78.

$P = 60^0 = \ell + 2r$ …(1)



Also $A = \dfrac{1}{2} = \ell r = \dfrac{1}{2}(60 - 2r)r$

$\Rightarrow A = 30r - r^2$

$\Rightarrow \dfrac{dA}{dr} = 30 - 2r$ and $\dfrac{d^2A}{dr^2} = -2$

$\therefore\ \dfrac{dA}{dr} = 0$

$\Rightarrow\ \ell = 15$

$\therefore$ Maximum Area for $\ell = 15$

79.

$x + y = 12$ .....(1)

$P = x^2 .(y)^4$

$\Rightarrow\ P = y^4(12 - y)^2$

$\Rightarrow\ \dfrac{dP}{dy} = y^4(2)(12 - y)(-1) + (12 - y)^2 . 4y^3$

$\therefore\ \dfrac{dP}{dy} = 0$

$\Rightarrow\ 2(12 - y). y^3 (24 - 2y - y) = 0$

$\Rightarrow\ 2(12 - y) . y^3 (24 - 3y) = 0$

$\Rightarrow\ y = 0$ or 12 or 8

$\Rightarrow\ y$ must be 8

$\Rightarrow$ Two parts will be 4 and 8

80.

$S = 2x + 3y;\ xy = 6$

$\Rightarrow\ S = 2x + 3\left(\dfrac{6}{x}\right) = 2x + \dfrac{18}{x}$

$\therefore\ \dfrac{dS}{dx} = 2 + 18\left(\dfrac{-1}{x^2}\right)$

$\therefore\ \dfrac{dS}{dx} = 0 \Rightarrow 1 = \dfrac{9}{x^2}$

$\Rightarrow\ x = 3$ or -3

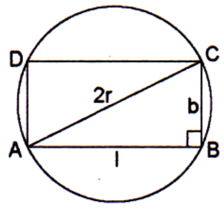
$\dfrac{d^2S}{dx^2} = \dfrac{36}{x^3}$ which > 0 for $x = 3$

$\Rightarrow\ x = 3, y = 2$

$\Rightarrow\ S_{min} = 2(3) + 3(2) = 12$

81.

Area of rectangle, $A = \ell b$ or $A = \ell.\sqrt{4r^2 - \ell^2}$



$\Rightarrow\ \dfrac{dA}{d\ell} = \dfrac{\ell(-2\ell)}{2\sqrt{4r^2 - \ell^2}} + \sqrt{4r^2 - \ell^2}$

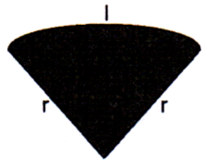
$= \dfrac{-\ell^2 + 4r^2 - \ell^2}{\sqrt{4r^2 - \ell^2}} = \dfrac{4r^2 - 2\ell^2}{\sqrt{4r^2 - \ell^2}}$

$\therefore\ \dfrac{dA}{d\ell} = 0$ $\quad\Rightarrow\ 4r^2 = 2\ell^2$

$\Rightarrow\ \ell^2 = 2r^2$ $\quad\therefore\ A = \sqrt{2}\, r\sqrt{2r^2} = 2r^2$

82.

$P = 2r + \ell = 2r + r\theta = (2 + \theta)r$



$A = \dfrac{\theta . r^2}{2} = \dfrac{\theta}{2}\left(\dfrac{p}{2+\theta}\right)^2$; p = constant

$\Rightarrow \dfrac{dA}{d\theta} = \dfrac{1}{2}p^2\left(\dfrac{\theta}{(2+\theta)^2}\right) = \dfrac{1}{2}p^2\left[\dfrac{(2+\theta)^2 - \theta.2(2+\theta)}{(2+\theta)^4}\right]$

$= \dfrac{1}{2}p^2\left[\dfrac{(2+\theta)(2+\theta-2\theta)}{(2+\theta)^4}\right] = \dfrac{1}{2}p^2 = \dfrac{(2-\theta)}{(2+\theta)^3} = 0$

$\Rightarrow \theta = 2^c$

83.

$f(x) = x^3 + bx^2 + ax + 5; x \in [1, 3]$

$c = \left(2 + \dfrac{1}{\sqrt{3}}\right)$, (a, b) =? and $f(1) = f(3)$

$\Rightarrow 1 + b + a + 5 = 27 + 9b + 3a + 5$

$\Rightarrow a + b + 1 = 3a + 9b + 27$

$\Rightarrow 2a + 8b = -26$

$\Rightarrow a + 4b = -13$ ...(1)

Now $f'(x) = 3x^2 + 2bx + a$

$f'(c) = 0$

$\Rightarrow 3c^2 + 2bc + a = 0$ ...(2)

$\because c = 2 + \dfrac{1}{\sqrt{3}}$ is a root of equation (2)

$\Rightarrow c = 2 - \dfrac{1}{\sqrt{3}}$ is also a root of equation (2)

$\Rightarrow \dfrac{-2b}{3} = 4;\ 4 - \dfrac{1}{3} = \dfrac{a}{3}$

$\Rightarrow a = 11, b = -6$

$\Rightarrow (a, b) = (11, -6)$

84.

(*a*) $f(x) = \tan x$ is discontinuous in $[0, \pi]$, having discontinuity at $\pi/2$

$\Rightarrow f(x) = \tan x$ can't be so.

(*b*) Next, $f(x) = \cos(1/x); x \in [-1, 1]$

$f(x)$ has discontinuity (*o*scillating at $x = 0$,

(*c*) $f(x) = x^2$ in $[2, 3]$

$f(2) = 4 \neq f(3) = 9$

(*d*) $f(x) = x(x + 3)\, e^{-x/2}$ in $[-3, 0]$

$f(-3) = f(0) = 0$,

$f(x)$ is continuous $\forall\ x \in [-3, 0]$,

Also $f(x)$ is differentiable $\forall\ x \in (-3, 0)$.

Hence Rolle's theorem is applicable

85.
Let $f(x) = ax^3 + bx^2 + cx$
$\Rightarrow$ $f(0) = f(1) = a + b + c = 0$
$\therefore$ By Rolle's Theorem, $f'(\alpha) = 0$ for at least one $\alpha \in (0, 1)$
$\Rightarrow$ $3ax^2 + 2bx + c = 0$ has at least one root in $(0, 1)$.

86.

$f(x) = \ell n\, x; x \in [1, 3]$
By L.M.V.T, $f'(c) = \dfrac{f(3) - f(1)}{3 - 1}$ for some $c \in (1, 3)$
$\Rightarrow$ $2f'(c) = \ell n\, 3 - \ell n\, 1 = \ell n\, 3$
$\Rightarrow$ $f'(c) = \dfrac{1}{2} \ell og_e 3$

87.

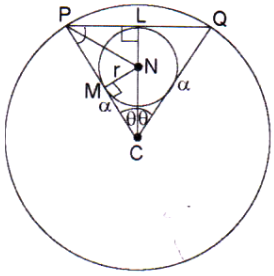
By L.M.V.T, $\exists\, c \in (0, 2)$ such that $f'(c) = \dfrac{f(2) - f(0)}{2 - 0}$
i.e., $f'(c) = \dfrac{f(2)}{2}$
$\because$ $|f'(x)| \le 1/2\ \forall\ x \in [0, 2]$
$\Rightarrow$ $|f'(c)| \le \dfrac{1}{2}$
$\Rightarrow$ $\left|\dfrac{f(2)}{2}\right| \le \dfrac{1}{2}$
$\Rightarrow$ $|f(2)| \le 1$
Similarly applying L.M.V.T on $[0, x]$; where $x \in (0, 2]$,
we see $|f(x)| \le 1\ \forall\ x \in (0, 2]$
Also $|f(0)| = 0$
$\Rightarrow$ $|f(x)| \le 1\ \forall\ x \in [0, 2]$

88.

$f(0) = 2g(0) = 0, f(1) = 6$
Let $F(x) = f(x) = 2g(x)$
$\Rightarrow$ $F(x)$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$.
$\therefore$ By L.M.V.T $\exists\, c \in (0, 1)$
Such that $F'(c) = \dfrac{F(1) - F(0)}{1 - 0}$ i.e., $f'(c) - 2g'(c) = [f(1) - 2g(1)] - [f(0) - 2g(0)]$
$\Rightarrow$ $0 = 6 - 2g(1) - 2 + 0 \Rightarrow g(1) = 2$

89.

In ΔPCN, $PM + MC = \alpha$



$\Rightarrow r\cot\left(\frac{\pi}{4}-\frac{\theta}{2}\right) + r\cot\theta = \alpha$

$\Rightarrow r\left[\cot\left(\frac{\pi}{4}-\frac{\theta}{2}\right)+\cot\theta\right]=\alpha$

$\Rightarrow r\left[\frac{\cot\frac{\theta}{2}+1}{\cot\frac{\theta}{2}-1}+\frac{\cot\theta}{1}\right]=\alpha$

$\Rightarrow r\left[\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}+\frac{\cot\theta}{1}\right]=\alpha$

$\Rightarrow r\left[\frac{1+\sin\theta}{\cos\theta}+\cot\theta\right]=\alpha$

$\Rightarrow r[\sec\theta + \tan\theta + \cot\theta] = \alpha$

$\Rightarrow r = \frac{\alpha}{\sec\theta+\tan\theta+\cot\theta}$

$\Rightarrow r = \frac{\alpha}{\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)}$

$\Rightarrow r = \frac{\alpha\sin\theta\cos\theta}{(\sin\theta+1)}$

$\Rightarrow r = \frac{\alpha}{2}.\frac{\sin 2\theta}{(1+\sin\theta)}$

$\Rightarrow \frac{dr}{d\theta}=\frac{\alpha}{2}.\frac{(1+\sin\theta)(2\cos 2\theta)-\sin 2\theta\cos\theta}{(1+\sin\theta)^2}$

$\therefore \frac{dr}{d\theta} = 0$

$\Rightarrow (1 + \sin\theta)(2 - 4\sin^2\theta) - 2\sin\theta\cos^2\theta$

$\Rightarrow (1 + \sin\theta)[2 - 4\sin^2\theta - 2\sin\theta(1 - \sin\theta)]$

$\Rightarrow (1 + \sin\theta)(-2\sin^2\theta - 2\sin\theta + 2) = 0$

$\Rightarrow \sin\theta = -1$ or $\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-1\pm\sqrt{1+4}}{2}$

$\Rightarrow \sin\theta = \frac{\sqrt{5}-1}{2}$

90.

$\Delta = \frac{1}{2}$ ab sin θ

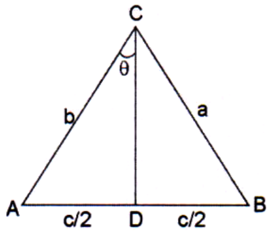
$\Rightarrow \frac{d\Delta}{d\theta} = \frac{1}{2} ab \cos\theta$

∴ For maximum area, $\frac{d\Delta}{d\theta} = 0$

$\Rightarrow$ cos θ = 0

$\Rightarrow$ θ = π/2

$\Rightarrow$ Δ *ABC* will be a right ∠*d* Δ, with ∠*C* = π/2



$\Rightarrow a^2 + b^2 = c^2$ …..(1)

Also, length median AD = $\frac{\sqrt{2a^2 + 2b^2 - c^2}}{2} = \frac{\sqrt{c^2}}{2} = \frac{c}{2}$

$\Rightarrow$ AD = $\frac{\sqrt{a^2 + b^2}}{2}$